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2014 **Taiwan Selection Test for PMWC and EMIC**

Preliminary Round Paper II Answer

1. The sum of the two digits of a number is 8. When the digits are written down in the reverse order, the new number is 18 more than the old number. What is the old number?

【Solution】

The old number can only be 17, 26 or 35. Since $71-17>62-26>53-35=18$, the old number is 35.

Answer：35

2. As a reward for winning a track meet, each of the 9 team members receives an award of 1500 dollars. The coach also receives an award which is 900 dollars more than the average award for the 10 of them. How much is the coach's award?

【Solution】

If the coach lets the team members share the extra 900 dollars, each will get 100 dollars more and now all 10 people will have the same amount of award money. Since the award for a team member is 1500 dollars, everyone now has 1600 dollars. Hence the coach's award is 1600+900=2500 dollars.

Answer：2500 dollars

3. In a party with more than 30 but less than 40 children, each boy shakes hands with 2 girls while each girl shakes hands with 3 boys. How many boys are at the party?

【Solution】

The ratio of the number of boys to the number of girls is 3:2. Hence the total number of children is a multiple of 5. Strictly between 30 and 40, the only multiple of 5 is 35. Hence the number of boys is $35\times\frac{3}{3+2}=21$.

Answer：21 boys

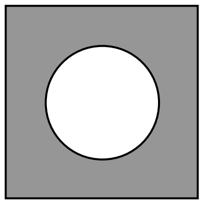
4. Beef sausages cost 150 dollars per kilogram while pork sausages cost 200 dollars per kilogram. A housewife spends all of her money and buys an equal number of beef sausages and pork sausages. If she spends half of her money on beef sausages and the other half on pork sausages, she will get 2 more kilograms of sausages. How much money does she have?

【Solution】

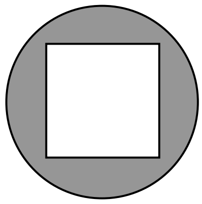
Note that 150 : 200 = 3 : 4 and the least common multiple of 150 and 200 is 600. The housewife spends $\frac{4}{7}$ of her money on pork sausages. The change in plan will transfer $\frac{4}{7}-\frac{1}{2}=\frac{1}{14}$ of her money from the pork sausage fund to the beef sausage fund. If the amount of the transfer is 600 dollars, she will get 3 kilograms less of pork sausages and 4 kilograms more of beef sausages. Since the actual gain in weight is 2 kilograms, the amount of the transfer must be doubled to 1200 dollars. Hence the housewife has $1200\div\frac{1}{14}=16800$ dollars.

Answer：16800 dollars

5. When viewed from the side, a solid appears as shown in the diagram below on the left. When viewed from above, it appears as shown on the right. Make a sketch of this solid.



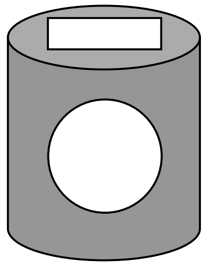
Viewed from the side



Viewed from above

【Solution】

The view from above shows that the solid is a cylinder with a vertical square hole at its centre. The view from the side shows that there is a horizontal round hole through the cylinder. A sketch is shown in the diagram.



6. In how many ways can 20 beads be distributed into 8 boxes so that each box contains an odd number of beads, if the beads are indistinguishable and the boxes are indistinguishable?

【Solution】

It is easy to see that the number of boxes each with only one bead is at least 2. We consider six cases.

Case 1. There are exactly 2 boxes each with only one bead.
The only possible distribution is 3+3+3+3+3+3+1+1.

Case 2. There are exactly 3 boxes each with only one bead.
We must move 2 beads from one box to other boxes. However, since each box has an odd number of beads, the two beads must be moved to the same box. Hence the only possible distribution is 5+3+3+3+3+1+1+1.

Case 3. There are exactly 4 boxes each with only one bead.
Another 2 beads must be moved from a box with 3 beads, and these 2 beads must stay together. Since they can either go to the box with 5 beads or another box with 3 beads, there are two possible distributions. They are 7+3+3+3+1+1+1+1 and 5+5+3+3+1+1+1+1.

Case 4. There are exactly 5 boxes each with only one bead.
Another 2 beads must move, resulting in three possible distributions. They are 9+3+3+1+1+1+1+1, 7+5+3+1+1+1+1+1 and 5+5+5+1+1+1+1+1.

Case 5. There are exactly 6 boxes each with only one bead.
Another 2 beads must move, resulting in three possible distributions. They are 11+3+1+1+1+1+1+1, 9+5+1+1+1+1+1+1 and 7+7+1+1+1+1+1+1.

Case 6. There are exactly 7 boxes each with only one bead.
The only possible distribution is 13+1+1+1+1+1+1+1.

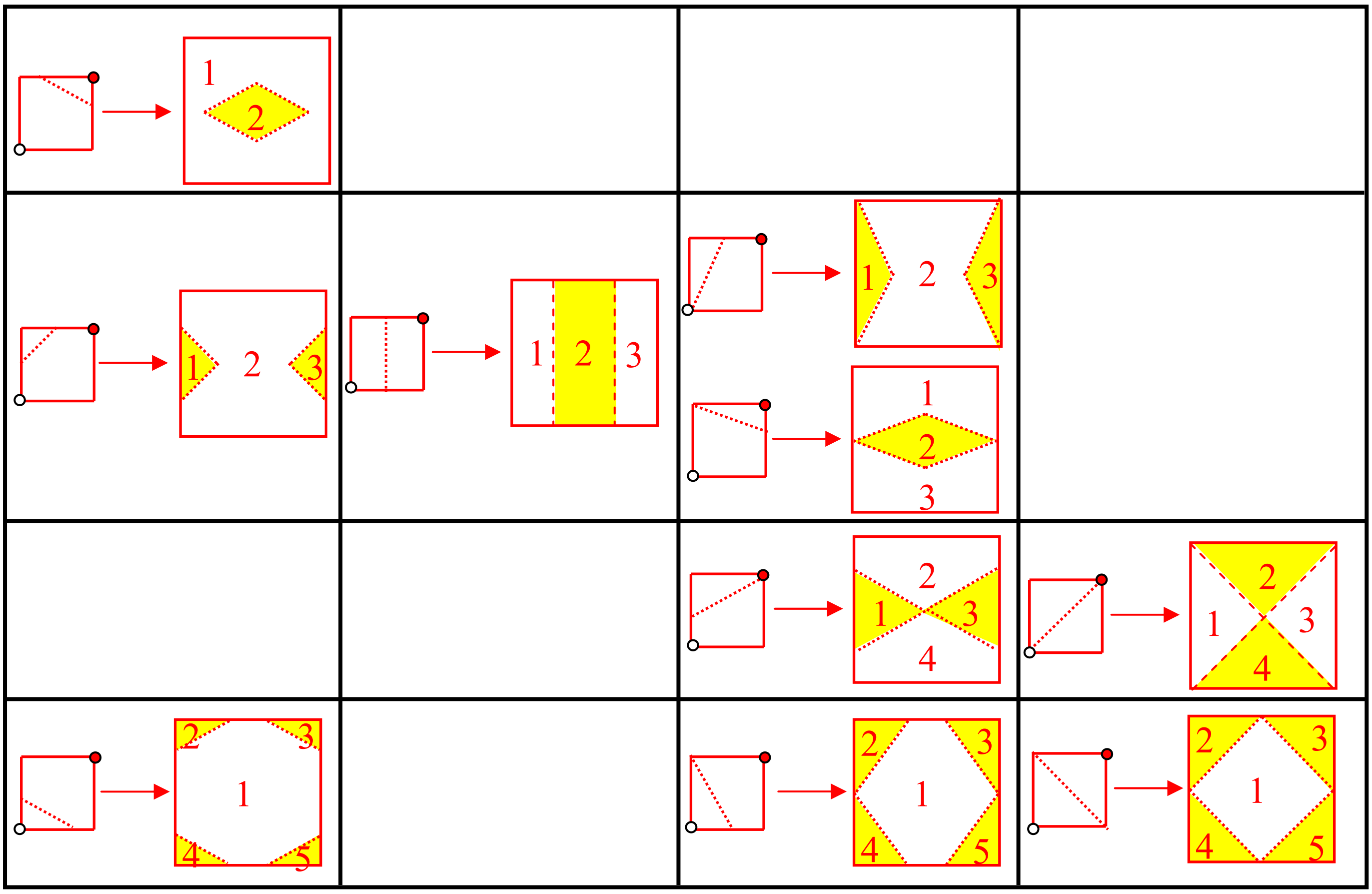
The total number of possible distributions is 1+1+2+3+3+1=11.

Answer：11

7. A 2×2 piece of paper is folded twice into a 1×1 stack of thickness 4. A straight cut is made, and the stack is unfolded into a number of pieces. What are the possible numbers of pieces?

【Solution】

In each 1×1 stack shown in the diagram below, the red dot marks the centre of the original 2×2 square and the white dot marks where the four corners of the original square come together. In the first column, the cut connects two adjacent sides of the stack. In the second column, the cut connects two opposite sides of the stack. In the third column, the cut passes through one corner of the stack. In the fourth column, the cut passes through two corners of the stack. The number of pieces may be 2, 3, 4 or 5, shown in different rows.



Answer：4

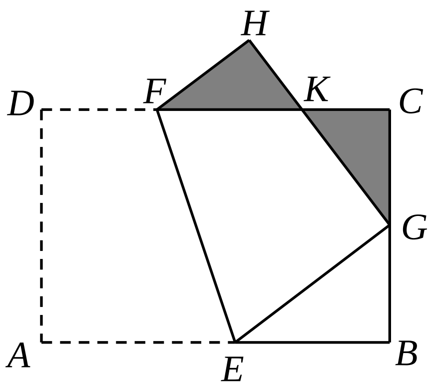
8. In a certain year, a particular day in each month is not a Sunday. What is that day of the month?

【Solution】

Let the days of the week be numbered 0, 1, 2, 3, 4, 5 and 6 in cyclic order. Consider the weekday number of the 1-st day of the month. In going from one month to the next, ignoring January and February for now, this number goes up by 3 if the preceding month has 31 days, and by 2 if the preceding month has only 30 days. Of course, if this number increases beyond 6, we subtract 7. Let the weekday number be 0 for March 1. Then it is 3 for April 1, 5 for May 1, 1 for June 1, 3 for July 1, 6 for August 1, 2 for September 1, 4 for October 1, 0 for November 1 and 2 for December 1. We have seen all of 0, 1, 2, 3, 4, 5 and 6. Hence the day we seek is not the 1-st day of the month. It then follows that it cannot be any of the 2-nd to the 30-th days either. Still taking the weekday number of March 1 to be 0, March 31 will be 2, May 31 will be 2+3+2=7 which becomes 0, July 31 will be 0+3+2=5, August 31 will be 5+3=8 which becomes 1. October 31 will be 1+3+2=6. December 31 will be 6+3+2=11 which becomes 4. In a normal year, January 31 will be 0 − 1 = −1 which becomes 6. In a leap year, January 1 will become 0 − 2 = − 2 which becomes 5. It follows that if 3 stands for Sunday, then the 31-st day of those months with that many days will not fall on a Sunday.

Answer：the 31-st day

9. A rectangular piece of paper $ABCD$ with AD = 12 cm is folded along the segment EF, with E on AB and F on CD, such that A lands on the midpoint G of BC and D lands on a point H outside $ABCD$. K is the point of intersection of CF and HG. If the triangles FHK and GCK are congruent, what is the area, in cm^2, of $ABCD$?



【Solution】

We have $DF = FH = GC = 6\,\text{cm}$ and $FC = GH = AD = 12\,\text{cm}$. It follows that $CD = DF + FC = 18\,\text{cm}$ so that the area of $ABCD$ is $AD \times CD = 216\,\text{cm}^2$.

Answer：216 cm^2

10. A 8056-digit number consisting of 2014 copies of 2014 written end-to-end. There is a remainder when it is divided by 11. What is the last digit of the quotient?

【Solution】

By the Test of Divisibility for 11, a number leaves the same remainder as its alternate digit-sum when divided by 11. The alternate digit-sum is (−2 + 0 − 1 + 4) + (−2 + 0 − 1 + 4) +…+ (−2 + 0 − 1 + 4) = 2014, and the remainder is 1 when 2014 is divided by 11. Hence the 8056-digit number 20142014…2013 is divisible by 11. The last digit of the quotient is clearly 3.

Answer：3

11. Each square of a 4×4 table contains either + or −. The four corner squares of any 4×4, 3×3 or 2×2 sub-table contain two +s and two −s. Find all such tables up to symmetry.

【Solution】

Label the squares as shown in the diagram below on the left. Two of A, D, M and P are +s and the other two are −s. We consider two cases.

Case 1. The two +s are at opposite corners, say A and P.

Then D and M contain −s. Now two of C, I and K contain −s, so that one of C and I must contain a −. By symmetry, we may assume that I contains a −s. This forces the signs in each of the remaining squares in the order (J, N), (K, O), L, (E, G), (F, H), B and C. The completed table is shown in the diagram below in the middle.

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

+	−	+	−
+	−	+	−
−	+	−	+
−	+	−	+

+	−	+	−
−	+	−	+
−	+	−	+
+	−	+	−

Case 2. The two +s are at adjacent corners, say A and M.

Then D and P contain −s. Now E and I cannot both contain +s. By symmetry, we may assume that E contains a −. If G contains a +, then O contains a −. Hence both K and L contain +s, and we have three +s among G, H, K and L. Hence G must contain a −. This forces the signs in each of the remaining squares in the order (C, H), O, (I, K), L, (F, J), B and N. The completed table is shown in the diagram above on the right.

12. Three black markers and three white markers placed alternately in six adjacent squares somewhere in an infinite row. In each move, you may shift two adjacent markers to two adjacent vacant squares, without reversing the order of the two markers. What is the minimum number of moves to make the markers occupy six adjacent squares again, but with all three white markers to the left of all three black markers?

【Solution】

Number the markers 1, 2, 3, 4, 5 and 6 from the left, so that the black markers are numbered 1, 3 and 5. Eventually, marker 2 must move to the left of marker 1, marker 4 to the left of marker 3, and 6 to the left of 5. Since each move can accomplish only one of these objectives, the number of moves cannot be less than three. The minimum number is in fact 3, as shown by the sequence of 3 moves in the diagram below.

		●	○	●	○	●	○						
		●	○	●			○	○	●				
				●	●	○	○	○	●				
						○	○	○	●	●	●		

Answer：3 moves