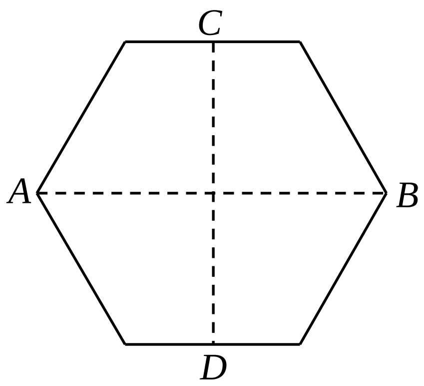
2015 Taiwan Selection Test for PMWC and EMIC

Final Round Paper I (Time Allowed : 90 Minutes)

- Write down all answers on the answer sheet. Each problem is worth 10 points and the total is 120 points.

1. A and B are opposite vertices of regular hexagon. C and D are midpoints of opposite sides such that CD is perpendicular to AB. The area of the hexagon is 126 cm^2. What is the area, in cm^2, of the rectangle with length AB and width CD?



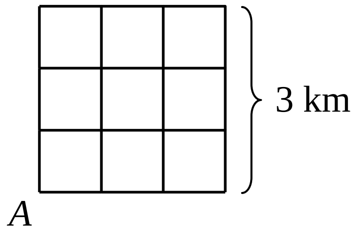
2. The six-digit number $\overline{17A32B}$ is a multiple of 88. What is the maximum value of the quotient when this number is divided by 88?

3. The weight of each of five coins has three possible values. However, there are only two different values among the weights of the five coins. In how many different ways can this happen?

4. Going with the current, a ship takes 6 hours to get from A to B. Going against the current, the ship takes 7 hours to get from B back to A. The speeds of the ship and of the current do not change. In how many hours will a raft flow downstream from A to B?

5. A garden is in the shape of a right triangle. The length of a side of the right angle is 35 m. The lengths, in m, of the other two sides are also positive integers. What is the minimum value of the perimeter, in m, of this garden?

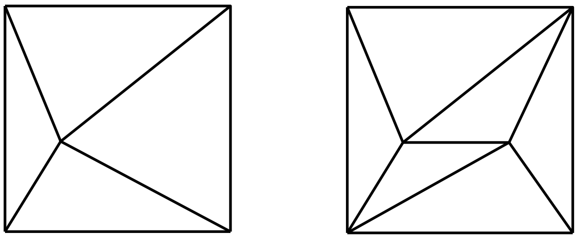
6. The diagram shows a small town with 9 blocks each measuring 1 km by 1 km. Start from a corner of the town, a man sweeps every section of the streets and returns to his starting point. He may walk along sections he has already swept. What is the minimum distance, in km, he has to move?



7. The factorial of a positive integer n, denoted by $n!$, is the product of all positive integers from 1 to n inclusive. Thus $5!=1\times2\times3\times4\times5$. Find the largest three-digit number which is equal to the sum of the factorials of its three digits.

8. The surface area of a box without a lid is 108 cm^2. What is the maximum value of its volume, in cm^3?

9. Inside a square are 15 points. Some pairs of these points are joined by line segments, and some of these points are joined to some of the vertices of the squares by line segments. All line segments do not intersect except at their endpoints. They divide the square into regions each of which is bounded by exactly three segments. How many triangles are there? The diagram below on the left shows that if there is only 1 point inside the triangle, then there are 4 triangles. The diagram below on the right shows that if there 2 points inside the square, then there are 6 triangles.



10. In a chess tournament, each participant plays a game against every other participant. Two of them are Grade 7 students while the remaining ones are all Grade 8 students. A win is worth 2 points, a draw 1 point and a loss 0 points. Between them, the two Grade 7 students score 16 points, while each Grade 8 student has the same score as one another. What is the maximum number of Grade 8 student in this tournament?

11. Each of the first two terms of a sequence is 59. Starting from the third term, each is the sum of the preceding two terms. What is the remainder when the 2015th term of this sequence is divided by 3?

12. Start with a square piece of paper. In the first move, use a straight cut to divide it into two pieces. In each subsequent move, use a straight cut to divide any of the pieces into two pieces. What is the minimum number of moves required in order to obtain at least five 15-sided polygons among the pieces?

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Final Round Paper II (Time Allowed : 60 Minutes)

- Each question is worth 25 marks for a maximum score of 100 marks. Fill in the answers in the space provided. Detailed solutions to Problems 2 and 4 are required.

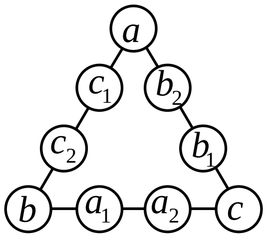
1. Dissect a square into 9 rectangles such that no two adjacent rectangles can be combined into a single rectangle by erasing their common border.

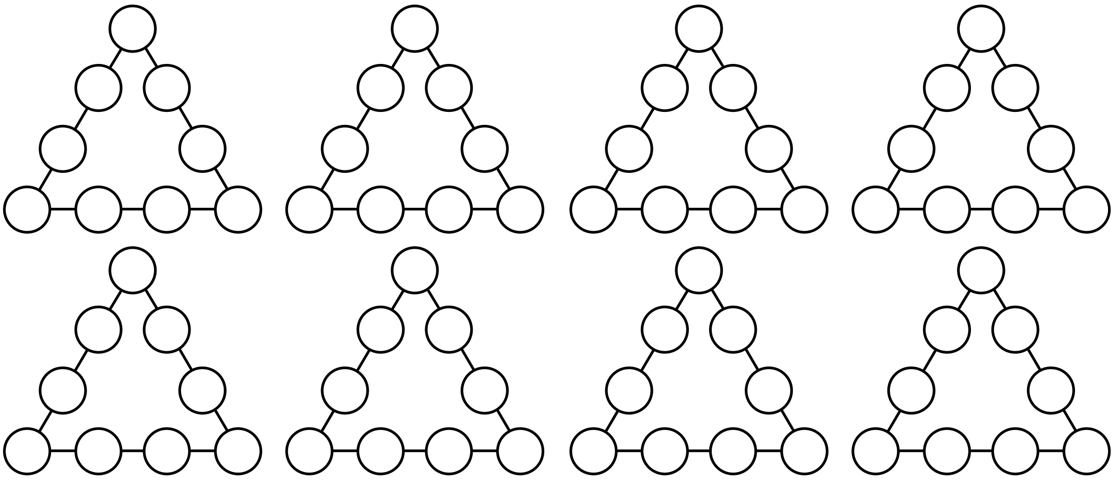
Answer：____________________

2. There are at least 30 real coins among 31 coins, and the last may be real or fake. All real coins weigh the same. A fake coin is either heavier or lighter than a real coin. We wish to determine whether there is a fake coin, and if so, whether it is heavier or lighter. What is the minimum number of weighings required on an standard balance?

Answer： _______________ weighings

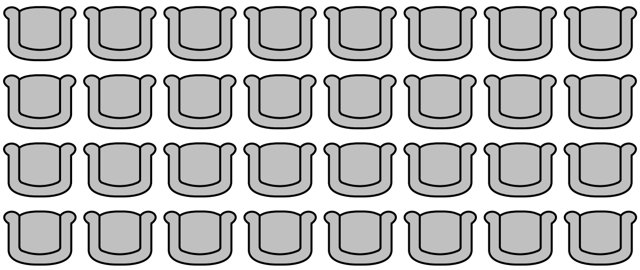
3. In the diagram below, replace each of a, a_1, a_2, b, b_1, b_2, c, c_1 and c_2 with a different one of the numbers from 1 to 9, so that $a>b>c$, $a_1>a_2$, $b_1>b_2$, $c_1>c_2$ and $b+a_1+a_2+c=c+b_1+b_2+a=a+c_1+c_2+b=20$. Find all of different solutions.



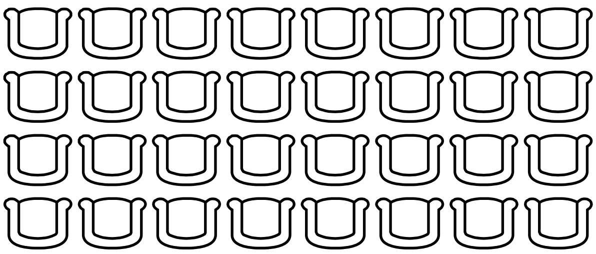


Answer：____________________

4. At a conference, 32 people are seated in 4 rows of 8. Each is either a Knight who always tells the truth, or a Knave who always lies. Each claims that there are at least one Knight and at least one Knave among the people occupying seats adjacent to his in the same row or the same column. What is the minimum number of Knaves at this conference?



Answer：The minimum number is____,



(Use “◎” to represent “Knave”)