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2015 小學數學競賽選拔賽初賽試題

第一試：計算題 (考試時間 1 小時)

◎請將答案填入答案卷對應題號的空格內，不須計算過程。答案若為分數請化為最簡分數。本題目卷正反面空白處可為作演算草稿紙。每題 5 分，共 100 分

- $97 \times 36 \div 25 + 97 \times 64 \div 25 = ?$
 $= 97 \times (36 + 64) \div 25 = 97 \times 100 \div 25 = 97 \times 4 = 388$
- $901 + 802 + 703 + 604 + \cdots + 109 = ?$
 $= (900 + 800 + 700 + \cdots + 200 + 100) + (1 + 2 + 3 + \cdots + 8 + 9) = 4500 + 45 = 4545$
- $5 \times 27 + 15 \times 9 + 45 \times 3 + 135 = ?$
 $= 135 + 135 + 135 + 135 = 135 \times 4 = 540$
- $9999 + 2015 + 2015 \times 9999 + 99 \div 99 = ?$
 $= 9999 + 99 \div 99 + 2015 \times (1 + 9999) = 10000 + 2015 \times 10000 = 20160000$
- $\frac{125}{4} \div 14 \times 196 \times \frac{8}{25} = ?$
 $= \frac{125}{4} \times \frac{8}{25} \times 196 \times \frac{1}{14} = \frac{1000}{100} \times 14 = 140$
- $100 + 99 + 98 - 97 - 96 - 95 + 94 + 93 + 92 - 91 - 90 - 89 + \cdots + 4 + 3 + 2 - 1 = ?$
 $= (100 + 99 + 98 - 97 - 96 - 95) + (94 + 93 + 92 - 91 - 90 - 89) + \cdots +$
 $(10 + 9 + 8 - 7 - 6 - 5) + (4 + 3 + 2 - 1) = \underbrace{9 + 9 + \cdots + 9}_{16\text{項}} + 8 = 9 \times 16 + 8 = 152$
- $(\frac{11}{6} + \frac{77}{30} - \frac{33}{10}) \times 101010 =$
 $= 11 \times (\frac{1}{6} + \frac{7}{30} - \frac{3}{10}) \times 101010 = 11 \times (\frac{5}{30} + \frac{7}{30} - \frac{3 \times 3}{30}) \times 101010$
 $= 11 \times \frac{(5 + 7 - 9)}{30} \times 101010 = 11 \times \frac{3}{30} \times 101010 = 11 \times 10101 = 111111$
- $\frac{11118888 + 33336666 + 55554444 + 77772222}{9999} = ?$
 $= \frac{10008 + 30006 + 50004 + 70002}{9} = \frac{160020}{9} = 17780$
- $2015 \times (20^2 \times 5 + 15) + 5 \times 45 - 4030 \times 15 = ?$
 $= 2015 \times 2015 + 15 \times 15 - 2 \times 2015 \times 15 = (2015 - 15)^2 = 4000000$

$$10. 0.8 + 9.8 + 99.8 + 999.8 + 9999.8 + \dots + 999999999.8 = ?$$

$$= (1 - 0.2) + (10 - 0.2) + (100 - 0.2) + \dots + (10^9 - 0.2) = 1111111111 - 2 = 1111111109$$

$$11. 785.123 + 758.132 + 875.213 + 857.231 + 578.312 + 587.321 = ?$$

$$= (5 + 7 + 8) \times 222 + (1 + 2 + 3) \times 0.222 = 4440 + 1.332 = 4441.332$$

$$12. 123454321 \times (1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1) \div 55555 = ?$$

$$= 11111^2 \times 5^2 \div 55555 = 55555$$

$$13. \frac{2015 + 2010 + 2005 + \dots + 15 + 10 + 5}{403^2 - 402^2 + 401^2 - 400^2 + \dots + 3^2 - 2^2 + 1} = ?$$

$$= \frac{5(403 + 402 + 401 + \dots + 3 + 2 + 1)}{(403^2 - 402^2) + (401^2 - 400^2) + \dots + (3^2 - 2^2) + 1}$$

$$= \frac{5(403 + 402 + 401 + \dots + 3 + 2 + 1)}{(403 + 402) + (401 + 400) + \dots + (3 + 2) + 1} = 5$$

$$14. 25 - \frac{9}{2} - \frac{17}{4} - \frac{33}{8} - \frac{65}{16} - \frac{129}{32} - \frac{257}{64} = ?$$

$$= 25 - 4\frac{1}{2} - 4\frac{1}{4} - 4\frac{1}{8} - 4\frac{1}{16} - 4\frac{1}{32} - 4\frac{1}{64} = 1 - (\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6})$$

$$= 1 - (\frac{2^5 + 2^4 + 2^3 + 2^2 + 2 + 1}{2^6}) = 1 - (\frac{2^6 - 1}{2^6}) = \frac{1}{64}$$

$$15. \frac{\frac{20152015}{1} - \frac{20152014}{1}}{\frac{20152014}{1} - \frac{20152015}{1}} = ?$$

$$= \frac{20152015^2 - 20152014^2}{20152014 \times 20152015 - 20152015 \times 20152014} = \frac{20152015^2 - 20152014^2}{20152014 \times 20152015 - 20152015 \times 20152014}$$

$$= \frac{(20152015 + 20152014)(20152015 - 20152014)}{1}$$

$$= 20152015 + 20152014 = 40304029$$

$$16. \frac{1599}{2015} \times (\frac{734}{2015} - \frac{462}{2015} + 1) - \frac{462}{2015} \times (\frac{734}{2015} - \frac{1599}{2015} + 1) - \frac{734}{2015} \times (\frac{1599}{2015} - \frac{462}{2015} + 1) = ?$$

$$= \frac{1599}{2015} \times \frac{734}{2015} - \frac{1599}{2015} \times \frac{462}{2015} + \frac{1599}{2015} - \frac{462}{2015} \times \frac{734}{2015} + \frac{462}{2015} \times \frac{1599}{2015} - \frac{462}{2015}$$

$$- \frac{734}{2015} \times \frac{1599}{2015} + \frac{734}{2015} \times \frac{462}{2015} - \frac{734}{2015}$$

$$= \frac{1599}{2015} - \frac{462}{2015} - \frac{734}{2015} = \frac{1599 - 462 - 734}{2015} = \frac{403}{2015} = \frac{1}{5} = 0.2$$

$$\begin{aligned}
 17. \quad & 5\frac{3}{15} + 2\frac{8}{35} + 1\frac{15}{63} + \frac{78}{99} + \frac{78}{143} = ? \\
 & = \frac{78}{3 \times 5} + \frac{78}{5 \times 7} + \frac{78}{7 \times 9} + \frac{78}{9 \times 11} + \frac{78}{11 \times 13} \\
 & = \frac{1}{2} \times 78 \times \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \frac{1}{9} - \frac{1}{11} + \frac{1}{11} - \frac{1}{13} \right) = \frac{1}{2} \times 78 \times \frac{10}{39} = 10
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 999999999^2 + 1999999999 = \\
 & = 999999999 \times (1000000000 - 1) + 1999999999 \\
 & = 999999999000000000 - 999999999 + 1999999999 \\
 & = 999999999000000000 + 1000000000 = 1000000000000000000 = 10^{18}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & 9999 \times \frac{\frac{1010}{11} \div \frac{20}{33}}{1.65 + 90 \frac{9}{11} \times 1 \frac{13}{20}} = ? \\
 & = 9999 \times \frac{\frac{1010}{11} \times \frac{33}{20}}{\frac{33}{20} + \frac{999}{11} \times \frac{33}{20}} = 9999 \times \frac{\frac{1010 \times 3}{20}}{(999 + 11) \times 3} = 9999 \times \frac{\frac{1010}{11}}{\frac{1010}{11}} = 9999
 \end{aligned}$$

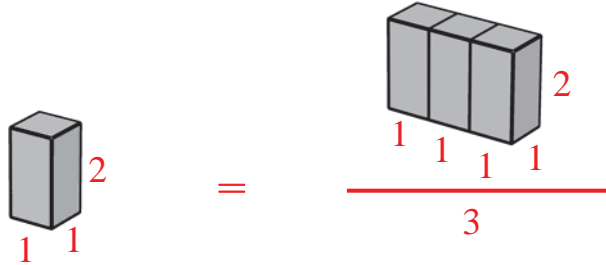
$$\begin{aligned}
 20. \quad & 1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + \dots + 99 \times 100 = ? \\
 & = 2 \times \left(\frac{1 \times 2}{2} + \frac{2 \times 3}{2} + \frac{3 \times 4}{2} + \frac{4 \times 5}{2} + \frac{5 \times 6}{2} + \dots + \frac{98 \times 99}{2} + \frac{99 \times 100}{2} \right) \\
 & = 2 \times \left[1 + (1+2) + (1+2+3) + (1+2+3+4) + \dots + (1+2+3+4+\dots+98) + \frac{99 \times 100}{2} \right] \\
 & = 2 \times \left\{ [1 + (1+2)] + [(1+2+3) + (1+2+3+4)] + \dots \right. \\
 & \quad \left. + [(1+2+3+4+\dots+97) + (1+2+3+4+\dots+98)] + \frac{99 \times 100}{2} \right\} \\
 & = 2 \times \left(2^2 + 4^2 + 6^2 + \dots + 98^2 + \frac{99 \times 100}{2} \right) \\
 & = 2 \times \left[4 \times (1^2 + 2^2 + 3^2 + \dots + 49^2) + \frac{99 \times 100}{2} \right] \\
 & = 2 \times \left(4 \times \frac{49 \times 50 \times 99}{6} + \frac{99 \times 100}{2} \right) \\
 & = 2 \times \left(\frac{98 \times 100 \times 99}{6} + \frac{99 \times 100}{2} \right) \\
 & = 99 \times 100 \times \left(\frac{98}{3} + 1 \right) \\
 & = \frac{99 \times 100 \times 101}{3} = 333300
 \end{aligned}$$

註：此題也可利用直接以下公式解出：

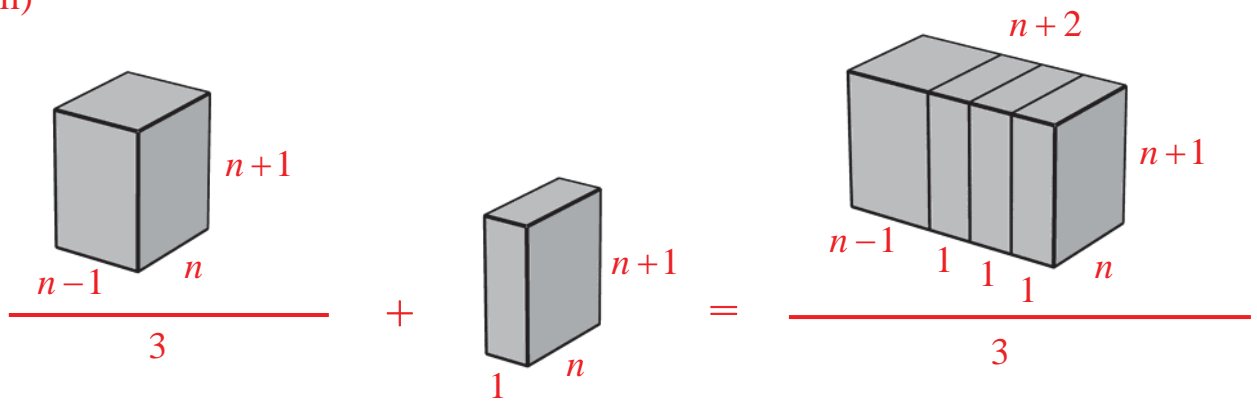
$$1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + \dots + (n-1) \times n = \frac{(n-1) \times n \times (n+1)}{3}$$

【證明 1】

(i)

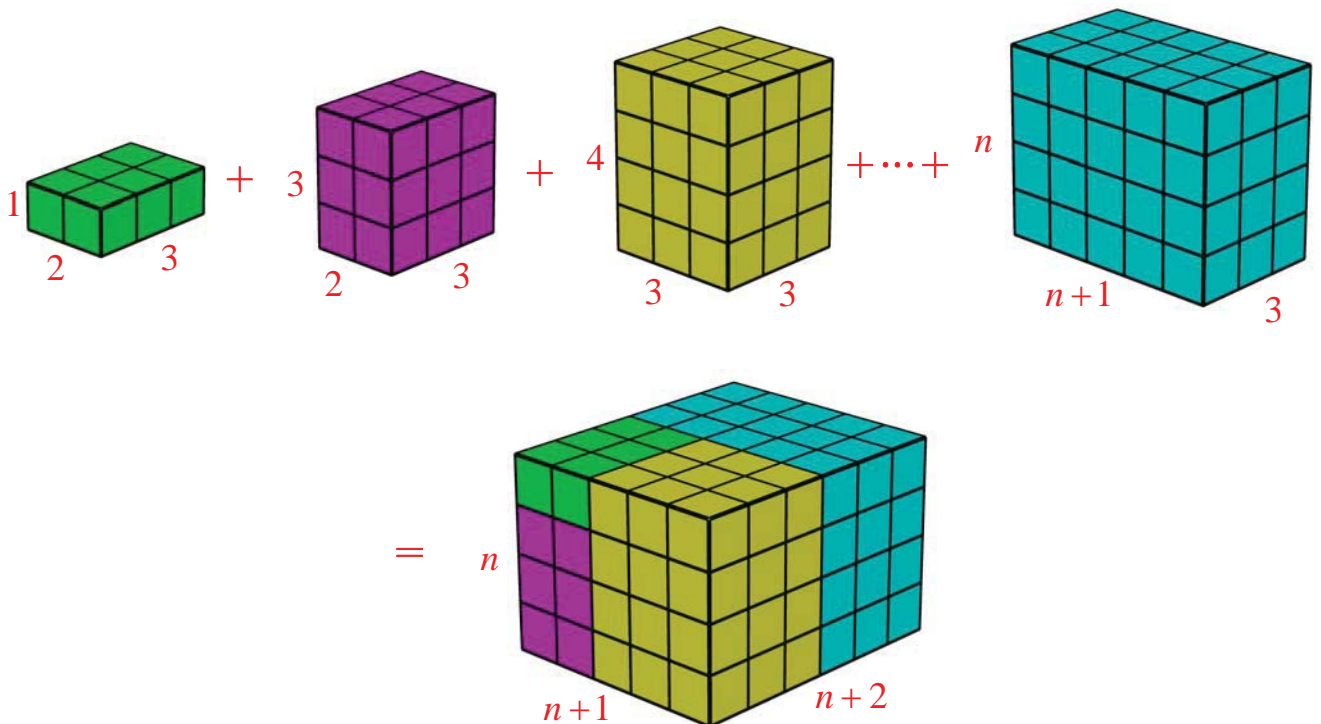


(ii)



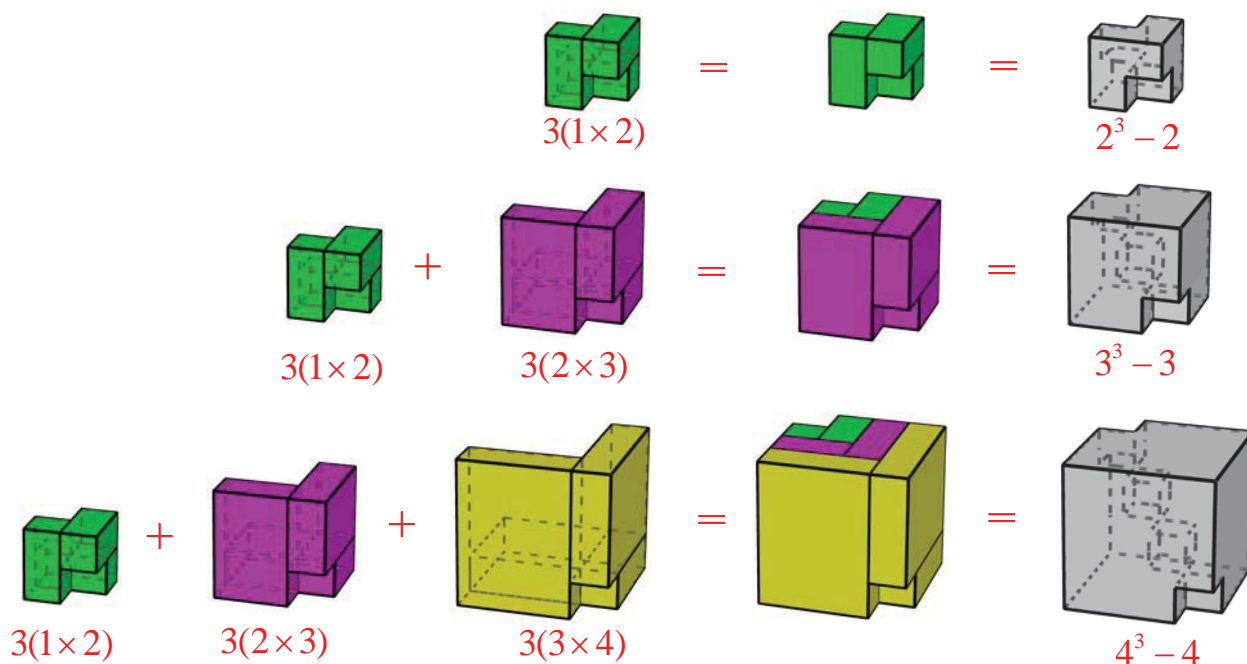
【證明 2】

此公式等價於 $3(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + \dots + n(n+1)) = n(n+1)(n+2)$:

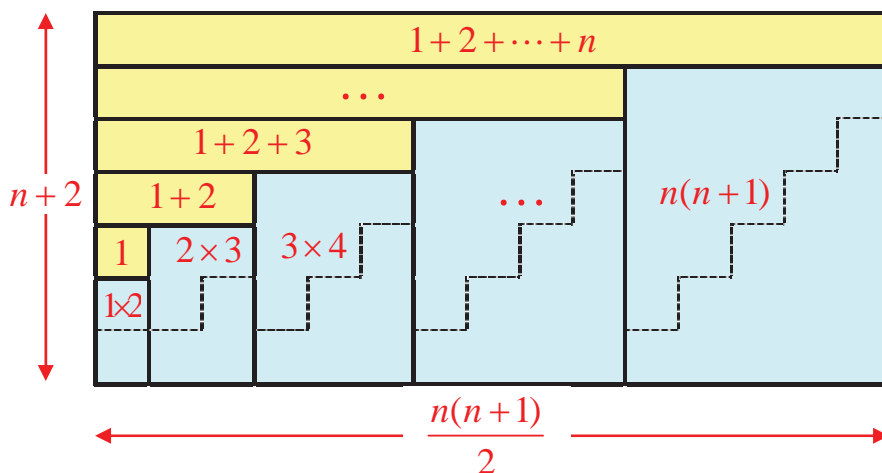


【證明 3】

此公式等價於 $3(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + \dots + (n-1) \times n) = n^3 - n$:



【證明 4】



令 $T_k = 1 + 2 + \dots + k$ ，則有

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) + (T_1 + T_2 + \dots + T_n) = \frac{n(n+1)(n+2)}{2}$$

而

$$T_1 + T_2 + \dots + T_n = \frac{1}{2}(1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1))$$

故有

$$\frac{3}{2}(1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1)) = \frac{n(n+1)(n+2)}{2}$$

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$