2015 Taiwan Selection Test for PMWC and EMIC
Preliminary Round Paper I (Time Allowed : 60 Minutes)

- **Write down all answers on the answer sheet.**
- **Each problem is worth 5 points and the total is 100 points.**

1. $97 \times 36 \div 25 + 97 \times 64 \div 25 = ?$

$= 97 \times (36 + 64) \div 25 = 97 \times 100 \div 25 = 97 \times 4 = 388$

2. $901 + 802 + 703 + 604 + \cdots + 109 = ?$

$= (900 + 800 + 700 + \cdots + 200 + 100) + (1 + 2 + 3 + \cdots + 8 + 9) = 4500 + 45 = 4545$

3. $5 \times 27 + 15 \times 9 + 45 \times 3 + 135 = ?$

$= 135 + 135 + 135 + 135 = 135 \times 4 = 540$

4. $9999 + 2015 + 2015 \times 9999 + 99 \div 99 = ?$

$= 9999 + 99 \div 99 + 2015 \times (1 + 9999) = 10000 + 2015 \times 10000 = 20160000$

5. $\dfrac{125}{4} \div 14 \times 196 \times \dfrac{8}{25} = ?$

$= \dfrac{125}{4} \times \dfrac{8}{25} \times 196 \times \dfrac{1}{14} = \dfrac{1000}{100} \times 14 = 140$

6. $100 + 99 + 98 - 97 - 96 - 95 + 94 + 93 + 92 - 91 - 90 - 89 + \cdots + 4 + 3 + 2 - 1 = ?$

$= (100 + 99 + 98 - 97 - 96 - 95) + (94 + 93 + 92 - 91 - 90 - 89) + \cdots +$

$(10 + 9 + 8 - 7 - 6 - 5) + (4 + 3 + 2 - 1) = \underbrace{9 + 9 + \cdots + 9}_{16\text{terms}} + 8 = 9 \times 16 + 8 = 152$

7. $(\dfrac{11}{6} + \dfrac{77}{30} - \dfrac{33}{10}) \times 101010 =$

$= 11 \times (\dfrac{1}{6} + \dfrac{7}{30} - \dfrac{3}{10}) \times 101010 = 11 \times (\dfrac{5}{30} + \dfrac{7}{30} - \dfrac{3 \times 3}{30}) \times 101010$

$= 11 \times \dfrac{(5 + 7 - 9)}{30} \times 101010 = 11 \times \dfrac{3}{30} \times 101010 = 11 \times 10101 = 111111$

8. $\dfrac{11118888 + 33336666 + 55554444 + 77772222}{9999} = ?$

$= \dfrac{10008 + 30006 + 50004 + 70002}{9} = \dfrac{160020}{9} = 17780$

9. $2015 \times (20^2 \times 5 + 15) + 5 \times 45 - 4030 \times 15 = ?$

$= 2015 \times 2015 + 15 \times 15 - 2 \times 2015 \times 15 = (2015 - 15)^2 = 4000000$

10. $0.8 + 9.8 + 99.8 + 999.8 + 9999.8 + \cdots + 999999999.8 = ?$

$= (1 - 0.2) + (10 - 0.2) + (100 - 0.2) + \cdots + (10^9 - 0.2) = 1111111111 - 2 = 1111111109$

11. $785.123 + 758.132 + 875.213 + 857.231 + 578.312 + 587.321 = ?$

$= (5 + 7 + 8) \times 222 + (1 + 2 + 3) \times 0.222 = 4440 + 1.332 = 4441.332$

12. $123454321 \times (1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1) \div 55555 = ?$

$= 11111^2 \times 5^2 \div 55555 = 55555$

13. $\dfrac{2015 + 2010 + 2005 + \cdots + 15 + 10 + 5}{403^2 - 402^2 + 401^2 - 400^2 + \cdots + 3^2 - 2^2 + 1} = ?$

$= \dfrac{5(403 + 402 + 401 + \cdots + 3 + 2 + 1)}{(403^2 - 402^2) + (401^2 - 400^2) + \cdots + (3^2 - 2^2) + 1}$

$= \dfrac{5(403 + 402 + 401 + \cdots + 3 + 2 + 1)}{(403 + 402) + (401 + 400) + \cdots + (3 + 2) + 1} = 5$

14. $25 - \dfrac{9}{2} - \dfrac{17}{4} - \dfrac{33}{8} - \dfrac{65}{16} - \dfrac{129}{32} - \dfrac{257}{64} = ?$

$= 25 - 4\dfrac{1}{2} - 4\dfrac{1}{4} - 4\dfrac{1}{8} - 4\dfrac{1}{16} - 4\dfrac{1}{32} - 4\dfrac{1}{64} = 1 - (\dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \dfrac{1}{2^4} + \dfrac{1}{2^5} + \dfrac{1}{2^6})$

$= 1 - (\dfrac{2^5 + 2^4 + 2^3 + 2^2 + 2 + 1}{2^6}) = 1 - (\dfrac{2^6 - 1}{2^6}) = \dfrac{1}{64}$

15. $\dfrac{\dfrac{20152015}{20152014} - \dfrac{20152014}{20152015}}{\dfrac{1}{20152014} - \dfrac{1}{20152015}} = ?$

$= \dfrac{\dfrac{20152015^2 - 20152014^2}{20152014 \times 20152015}}{\dfrac{20152015 - 20152014}{20152014 \times 20152015}} = \dfrac{20152015^2 - 20152014^2}{20152015 - 20152014}$

$= \dfrac{(20152015 + 20152014)(20152015 - 20152014)}{1}$

$= 20152015 + 20152014 = 40304029$

16. $\dfrac{1599}{2015} \times (\dfrac{734}{2015} - \dfrac{462}{2015} + 1) - \dfrac{462}{2015} \times (\dfrac{734}{2015} - \dfrac{1599}{2015} + 1) - \dfrac{734}{2015} \times (\dfrac{1599}{2015} - \dfrac{462}{2015} + 1) = ?$

$= \dfrac{1599}{2015} \times \dfrac{734}{2015} - \dfrac{1599}{2015} \times \dfrac{462}{2015} + \dfrac{1599}{2015} - \dfrac{462}{2015} \times \dfrac{734}{2015} + \dfrac{462}{2015} \times \dfrac{1599}{2015} - \dfrac{462}{2015}$

$- \dfrac{734}{2015} \times \dfrac{1599}{2015} + \dfrac{734}{2015} \times \dfrac{462}{2015} - \dfrac{734}{2015}$

$= \dfrac{1599}{2015} - \dfrac{462}{2015} - \dfrac{734}{2015} = \dfrac{1599 - 462 - 734}{2015} = \dfrac{403}{2015} = \dfrac{1}{5} = 0.2$

17. $5\dfrac{3}{15} + 2\dfrac{8}{35} + 1\dfrac{15}{63} + \dfrac{78}{99} + \dfrac{78}{143} = ?$

$= \dfrac{78}{3\times5} + \dfrac{78}{5\times7} + \dfrac{78}{7\times9} + \dfrac{78}{9\times11} + \dfrac{78}{11\times13}$

$= \dfrac{1}{2}\times78\times(\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{7}-\dfrac{1}{9}+\dfrac{1}{9}-\dfrac{1}{11}+\dfrac{1}{11}-\dfrac{1}{13}) = \dfrac{1}{2}\times78\times\dfrac{10}{39} = 10$

18. $999999999^2 + 1999999999 =$

$= 999999999\times(1000000000-1)+1999999999$

$= 999999999000000000 - 999999999 + 1999999999$

$= 999999999000000000 + 1000000000 = 1000000000000000000 = 10^{18}$

19. $9999\times\dfrac{\dfrac{1010}{11}\div\dfrac{20}{33}}{1.65+90\dfrac{9}{11}\times1\dfrac{13}{20}} = ?$

$= 9999\times\dfrac{\dfrac{1010}{11}\times\dfrac{33}{20}}{\dfrac{33}{20}+\dfrac{999}{11}\times\dfrac{33}{20}} = 9999\times\dfrac{\dfrac{1010\times3}{20}}{\dfrac{(999+11)\times3}{20}} = 9999\times\dfrac{\dfrac{1010}{11}}{\dfrac{1010}{11}} = 9999$

20. $1\times2+2\times3+3\times4+4\times5+5\times6+\cdots+99\times100 = ?$

$= 2\times(\dfrac{1\times2}{2}+\dfrac{2\times3}{2}+\dfrac{3\times4}{2}+\dfrac{4\times5}{2}+\dfrac{5\times6}{2}+\cdots+\dfrac{98\times99}{2}+\dfrac{99\times100}{2})$

$= 2\times[1+(1+2)+(1+2+3)+(1+2+3+4)+\cdots+(1+2+3+4+\cdots+98)+\dfrac{99\times100}{2}]$

$= 2\times\{[1+(1+2)]+[(1+2+3)+(1+2+3+4)]+\cdots$

$\quad\quad +[(1+2+3+4+\cdots+97)+(1+2+3+4+\cdots+98)]+\dfrac{99\times100}{2}\}$

$= 2\times(2^2+4^2+6^2+\cdots+98^2+\dfrac{99\times100}{2})$

$= 2\times[4\times(1^2+2^2+3^2+\cdots+49^2)+\dfrac{99\times100}{2}]$

$= 2\times(4\times\dfrac{49\times50\times99}{6}+\dfrac{99\times100}{2})$

$= 2\times(\dfrac{98\times100\times99}{6}+\dfrac{99\times100}{2})$

$= 99\times100\times(\dfrac{98}{3}+1)$

$= \dfrac{99\times100\times101}{3} = 333300$

Note：We can solve the problem by using the following formula:

$$1\times 2+2\times 3+3\times 4+4\times 5+5\times 6+\cdots+(n-1)\times n=\frac{(n-1)\times n\times(n+1)}{3}$$

【First Proof】

(i)
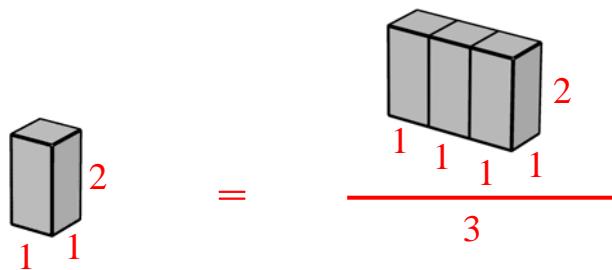

(ii)
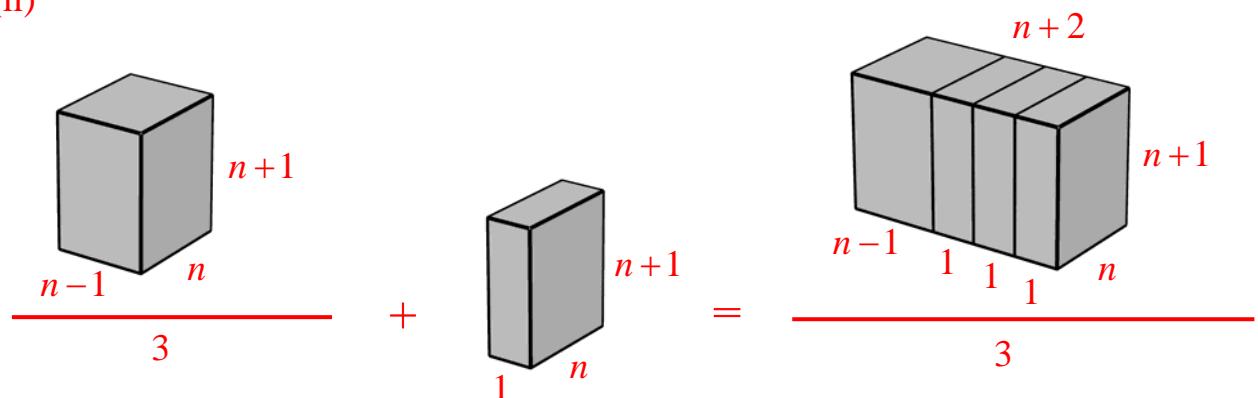
【Second Proof】

The formula is equivalent to

$$3(1\times 2+2\times 3+3\times 4+4\times 5+5\times 6+\cdots+n(n+1))=n(n+1)(n+2)：$$
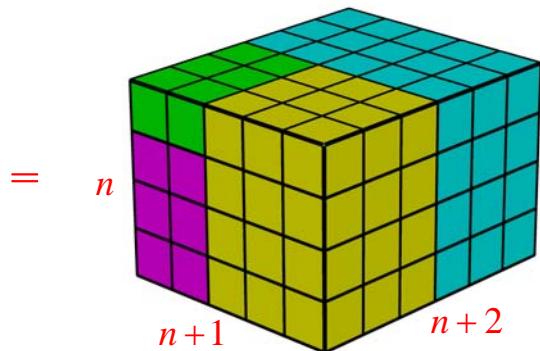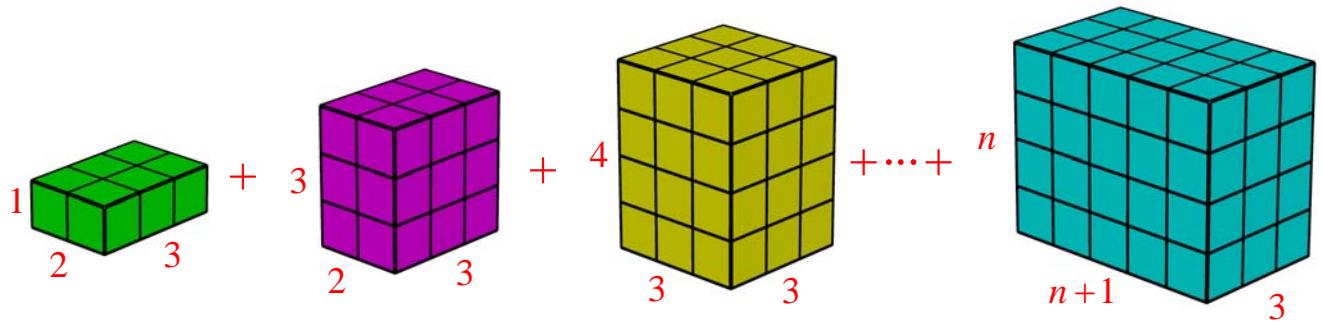
【Third Proof】

The formula is equivalent to

$$3(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + \cdots + (n-1) \times n) = n^3 - n :$$

$$3(1 \times 2) \qquad = \qquad \qquad = \qquad 2^3 - 2$$

$$3(1 \times 2) \quad + \quad 3(2 \times 3) \qquad = \qquad \qquad = \qquad 3^3 - 3$$

$$3(1 \times 2) \quad + \quad 3(2 \times 3) \quad + \quad 3(3 \times 4) \qquad = \qquad \qquad = \qquad 4^3 - 4$$
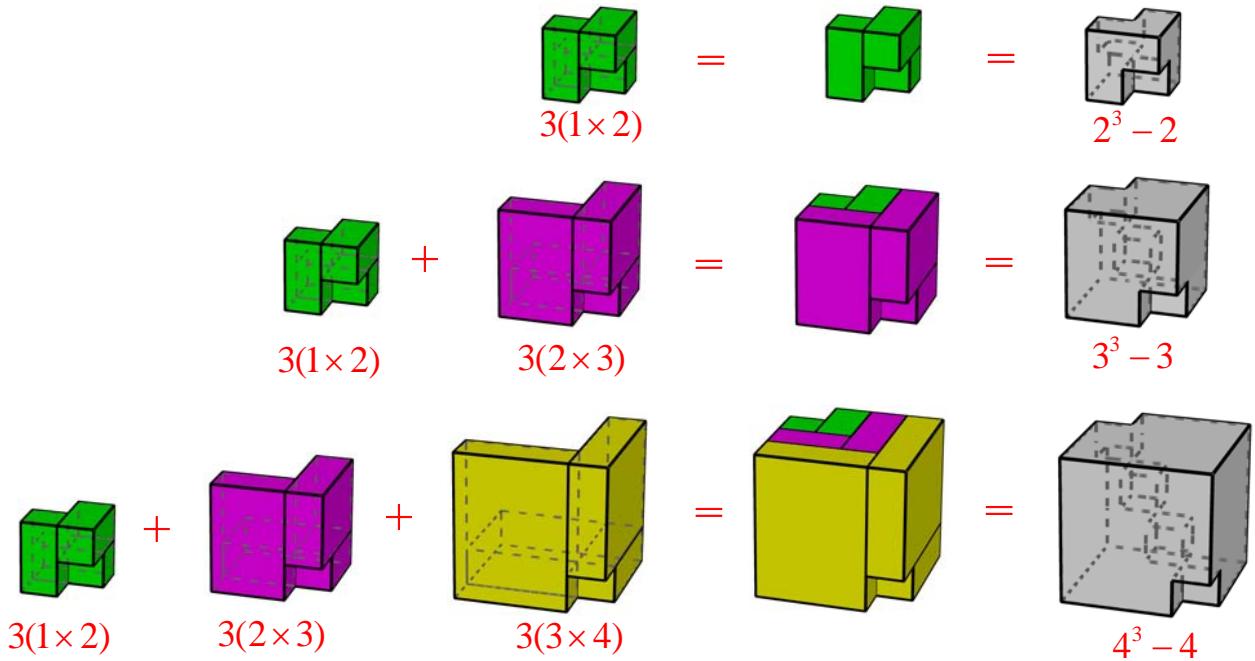
【Forth Proof】
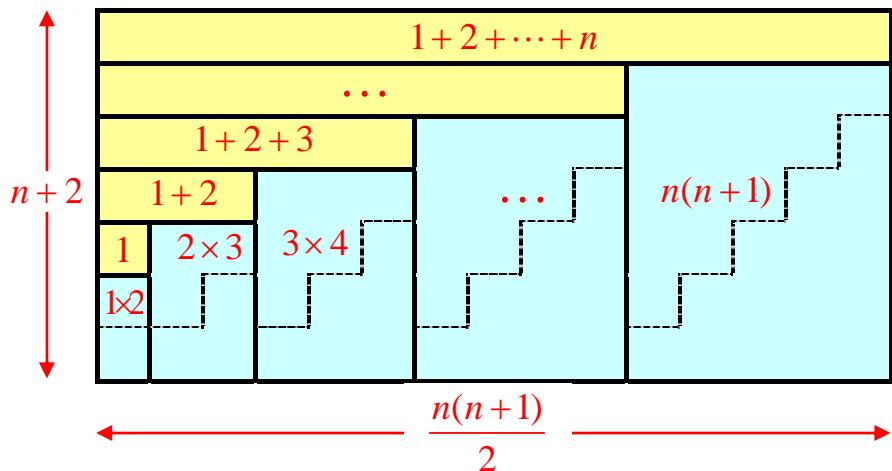
Let $T_k = 1 + 2 + \cdots + k$, then

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1) + (T_1 + T_2 + \cdots + T_n) = \frac{n(n+1)(n+2)}{2}.$$

Since

$$T_1 + T_2 + \cdots + T_n = \frac{1}{2}(1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1)),$$

we have

$$\frac{3}{2}(1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1)) = \frac{n(n+1)(n+2)}{2}$$

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$